Approximate Concepts Analysis Based on {1,0,-1}-valued Formal Contexts

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Abstract

With the rapid growth of data volume and the increasing complexity of data analysis, traditional formal concept analysis (FCA) models have become inadequate for addressing the diverse and intricate requirements of big data environments. In response to these challenges, this paper introduce a novel approximate concept model grounded in three-way concepts and incorporating the notion of {1,0,-1}-valued formal contexts. We formally introduce the foundational definitions of the proposed model, thoroughly examine its essential properties and the partial order relations among concepts, and explore its enhanced capacity for representing and reasoning about uncertain or {1,0,-1}-valued information. Furthermore, a comprehensive comparison with existing concept models is provided to highlight the advantages and distinguishing features of the proposed approach. Finally, we demonstrate the potential applications of this model in data analysis and knowledge representation, illustrating its value for addressing the complexities inherent in modern data-driven tasks.

Keywords

Formal concept analysis, Concept lattice, Approximate concepts.

1. INTRODUCTION

Formal concept analysis (FCA) is widely regarded as an effective approach for analyzing data that involves binary relationships between two types of elements: objects and attributes. These relationships are typically represented in what is called a formal context. Within this framework, the concept lattice serves as the primary data structure and plays a central role in organizing the information [1]. By systematically examining the associations present in the context, one can derive a collection of concepts, each described by a set of related objects and their shared attributes. The set of objects, often referred to as the extent, represents those instances that are linked by common features, while the intent is the corresponding set of attributes that are common across the extent. More precisely, the extent can be understood as the largest subset of objects that jointly exhibit a particular set of properties, and the intent as the maximal set of properties shared among all members of that object subset. These concepts, when arranged according to the inclusion relations between their extents or intents, form a partially ordered structure known as the concept lattice, which offers a hierarchical view of the data's underlying conceptual organization.

Over the years, formal concept analysis and the associated theory of concept lattices have attracted considerable academic attention. Researchers have explored a variety of topics, including the construction and expansion of concept lattices as well as their cognitive interpretations. These efforts have supported the development of FCA in multiple practical

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domains such as knowledge representation, reasoning mechanisms, data mining, and cognitive computing. A variety of algorithms have been proposed to facilitate the construction of concept lattices. For instance, Outrata et al. [3] enhanced the classical CbO algorithm by incorporating regularity checks, leading to the development of a more efficient variant known as FCbO. Similarly, Muangprathub J [5] introduced a novel approach aimed at accelerating the generation of concept lattices. Further contributions by Wisdom Lai et al. [6] addressed the challenges involved in the rapid updating of lattices, particularly in scenarios involving merging operations and inference rule adjustments. Despite these advancements, it is generally acknowledged that building concept lattices remains a computationally intensive task. As the number of objects within a formal context increases, the time complexity of lattice construction grows exponentially, reflecting the NP-hard nature of the problem. To address this, alternative strategies have been explored. For example, some researchers have investigated the idea of concept approximation. Cao Li et al. [8] proposed an approximation framework that preserves the underlying binary relations while simplifying the lattice structure. This line of work was extended by Wei Ling et al. [9] , who adapted the approach to decision contexts by introducing notions of strong and weak coordinateness, thereby offering flexibility for different decision-making scenarios. In addition, there have been efforts to bypass the need for a complete concept lattice altogether. Huilai Zhi et al. [10] proposed dividing the global formal context into smaller sub-contexts, each processed individually and later merged, which significantly reduces the computational burden by limiting the scale of each sub-lattice.

In traditional formal concept analysis, the interaction between objects and attributes is generally represented using binary values, typically indicating whether an object possesses a given attribute or not. In many practical scenarios, the association between objects and attributes involves degrees of uncertainty, partial information, or vagueness, which the classical binary formal context struggles to represent accurately. To address these limitations, researchers have proposed various generalized models, including {1,0,-1}-valued formal contexts [11], fuzzy formal contexts [12], etc. which offer greater flexibility and adaptability for complex and imprecise data environments.

These alternative formulations have also spurred the development of novel conceptual structures and extensions of the original concept lattice model. While classical concept lattices offer a solid foundation for analyzing relationships within information systems, their expressiveness is inherently limited—they can only capture the presence of attributes. To overcome this restriction and extend applicability, researchers have introduced more refined models. Notably, the integration of three-way decision theory has led to the emergence of three-way concept analysis [14], which enables simultaneous consideration of both commonly shared and commonly missing attributes among object sets. This line of work provides a more comprehensive understanding of attribute distribution within a data context. Building on these ideas, Qi et al. [17] proposed a new conceptual model grounded in three-valued logic and defined over a specially constructed three-valued formal context. Additionally, Wisdom Lai and colleagues [10, 18, 19] further advanced this direction by developing the three-way approximation concept, which has found applications in areas such as conflict analysis. These extensions have significantly broadened the applicability of concept lattice theory and have paved the way for its deeper integration into domains such as knowledge discovery, automated reasoning, and the resolution of inconsistencies in complex data systems.

This study puts forward a new variant of approximate three-way concepts, specifically designed for use within {1,0,-1}-valued formal contexts. The proposed model is structured around a set of objects and a corresponding tuple of attribute subsets. Unlike classical representations, this concept formulation incorporates more nuanced semantic layers, allowing for greater expressiveness in capturing the complexities inherent in real-world data.

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Consequently, it offers stronger capabilities in knowledge representation and demonstrates broader applicability in areas such as concept interpretation and data mining.

The structure of this paper is as follows. Section 2 introduces the necessary background and theoretical foundations. In Section 3, we define the new concept model and elaborate on its formal properties. Section 4 provides a comparative analysis between the proposed concept and traditional variants. Section 5 concludes the paper and outlines potential directions for future research.

2. PRELIMINARIES

A formal context describes the relationship between a set of objects and another set of attributes, which is a binary structure. Specifically, consider a non-empty set U representing the objects, a non-empty set V denoting the associated attributes, and to point out the mapping the any object and attribute, last element R represent that for any $x \in U$ and $a \in V$, if $R(x, a)=1$, it represents that object x support attribute a, if $R(x, a)=0$, it represents that object x does not support attribute a. The triple $K=(U, V, R)$ is called a formal context. The following contents can be found in the [1, 2].

Definition 1: A formal context as a triple $K=(U, V, R)$, with $X \in 2^U$ and $A \in 2^V$, the operator "*" on $2^U \rightarrow 2^V$ as well as on $2^V \rightarrow 2^U$ are defined as:

$2^U \rightarrow 2^V$: $X^* = \{a \in V \mid \forall x \in U, R(x, a)=1\}$

$2^V \rightarrow 2^U$: $A^* = \{x \in U \mid \forall a \in V, R(x, a)=1\}$

In triple (U, V, R), (X, A) is a formal concept, if there exists $X \in 2^U$ and $A \in 2^V$ satisfying $X^*=A$ and $A^*=X$, where X is the extent , means that something belong to this concept, dually, and A is the intent of the concept, which is the description.

Theorem 1: A formal context as a triple $K=(U, V, R)$, with $X, X_1, X_2 \in 2^U$ and $A, A_1, A_2 \in 2^V$, then the following property holds:

(1) Inclusion reverses under derivation: if $X_1 \subseteq X_2$ then $X_1^* \supseteq X_2^*$, dually, *if* $A_1 \subseteq A_2$, then$A_1^* \supseteq A_2^*$

(2) Applying the derivation operator twice yields a closure, i.e. $X \subseteq X^{**}$, $A \subseteq A^{**}$

(3) $X^* = X^{***}$, and $A^* = A^{***}$

(4) The image of a union equals the intersection of images: $(X_1 \cup X_2)^* = X_1^* \cap X_2^*$, and analogously on attribute sets. $(A_1 \cup A_2)^* = A_1^* \cap A_2^*$

(5) Conversely, the image of an intersection contains the union of images: $(X_1 \cap X_2)^* \supseteq X_1^* \cup X_2^*$, dually, $(A_1 \cap A_2)^* \supseteq A_1^* \cup A_2^*$

Definition 2: A formal context as a triple $K=(U, V, R)$, for which the partial order relation is defined for two concepts (X_1, A_1), (X_2, A_2) as: $(X_1, A_1) \leqslant (X_2, A_2) \Leftrightarrow X_1 \subseteq X_2, A_1 \supseteq A_2$.

The lattice structure of all the concepts through the partial order relation, the final lattice construct is called the concept lattice, and we denote it $L(K)$.

Definition 3: For any two concepts (X_1, A_1), (X_2, A_2) in the concept lattice, the supremum and the infimum are defined as respectively:

for supremum, we according the $(X_1, A_1) \vee (X_2, A_2)$ can obtain that $((X_1 \cup X_2)^{**}, A_1 \cap A_2)$,

for infimum, as well as based on $(X_1, A_1) \wedge (X_2, A_2)$ can obtain that $(X_1 \cap X_2, (A_1 \cup A_2)^{**})$.

In summary, a complete lattice can be constructed based on a formal context (U, V, R) to construct a complete lattice.

Example 1: $K=(U, V, R)$ is shown in Table 1, The concept lattice constructed from K is shown in Fig. 1.

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Table 1. formal context K

	a	b	c	d	e
1	1	0	0	1	0
2	0	1	1	0	0
3	0	1	0	0	1
4	1	1	1	1	0

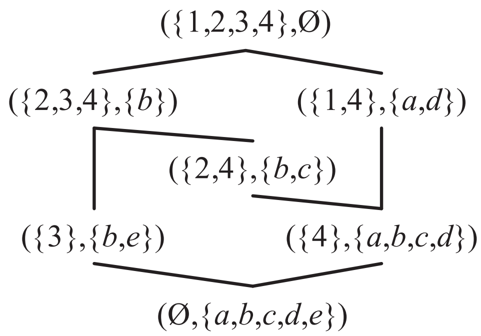

Figure 1. Concept lattice $L(K)$

For example, for concept ({2, 3, 4,}, {b}) the maximum common attribute set of object set {2, 3, 4} is {b}, dually, the maximum common object set of attribute set {b} is {2, 3, 4}.

3. APPROXIMATE THREE-WAY CONCEPTS

The binary relations in the classical formal context are either one or zero and can express a limited amount of information. In real world, the relationship between objects and attributes may be vague and indeterminate. To describe this, an indeterminate attribute is introduced on the basis of the formal context in Section 2.

Definition 4: To express uncertain or partial relationships between objects and attributes, we allow the binary relation, $R(x, a)$=1 to take values from the set {1, 0, -1}, Here, a value of 1 indicates that object x is associated with attribute a, while −1 signifies that the object clearly lacks this attribute. When $R(x, a)$ = 0, it reflects that the relationship between x and a is unknown or cannot be determined. In total, A data structure defined by a quadruple (U, V, {1, 0, -1}, R) is referred to as an {1,0,-1}-valued formal context.

By this method, the scope of formal context description information can be enhanced.

Definition 5: A {1,0,-1}-valued formal context as a quadruple K=(U, V, {1, 0, -1}, R), with $x \in U$, $a \in V$, and the positive operator "\$+", the negative operator "\$-", and the conflict operator "\$~" are defined respectively:

$x^{\$+} = \{a \in V \mid R(x, a)=1 \text{ or } R(x, a)=0\}$

$x^{\$-} = \{a \in V \mid R(x, a)=-1 \text{ or } R(x, a)=0\}$

$x^{\$\sim} = \{a \in V \mid R(x, a)=1 \text{ or } R(x, a)=-1\}$

Dually, the positive operator "\$+", the negative operator "\$-", and the conflict operator "\$~" on a $\rightarrow$ 2^U are defined as:

$a^{\$+} = \{x \in U \mid R(x, a)=1 \text{ or } R(x, a)=0\}$

$a^{\$-} = \{x \in U \mid R(x, a)=-1 \text{ or } R(x, a)=0\}$

$a^{\$\sim} = \{x \in U \mid R(x, a)=1 \text{ or } R(x, a)=-1\}$

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Definition 6: A {1,0,-1}-valued formal context as a quadruple K=(U, V, {1, 0, -1}, R), with $x \in U$, $a \in V$, and the positive operator "\$+", the negative operator "\$-", and the conflict operator "\$∼" are defined respectively:

In above, definition 6 defines a common property of a set of objects X based on Definition 5. In this way, an approximation concept can be further defined in definition 7.

Definition 7: A {1,0,-1}-valued formal context as a structure K=(U, V, {1, 0, -1}, R), with $X \in 2^U$ and $(A, B, C) \in 2^V \times 2^V \times 2^V$. The operators "▷" for $2^U \rightarrow 2^V \times 2^V \times 2^V$ and "◁" for $2^V \times 2^V \times 2^V \rightarrow 2^U$ are defined as follow, respectively:

$X^{\triangleright} = (X^{\$+}, X^{\$-}, X^{\$\sim})$

$(A, B, C)^{\triangleleft} = \{x \in U \mid (x^{\$+} \supseteq A) \wedge (x^{\$-} \supseteq A) \wedge (x^{\$\sim} \supseteq A)\}$

If $X^{\triangleright} = (A, B, C)$ and $(A, B, C)^{\triangleleft} = X$, then $(X, (A, B, C))$ is said to be an approximate concept, where X is the extent of the approximation concept and (A, B, C) is the intent of the approximation concept.

Definition 8: A {1,0,-1}-valued formal context as a quadruple K=(U, V, {1, 0, -1}, R),the partial order relation between approximate concepts $(X_1, (A_1, B_1, C_1))$, $(X_2, (A_2, B_2, C_2))$ is defined as:

$(X_1, (A_1, B_1, C_1)) \leqslant (X_1, (A_1, B_1, C_1)) \Leftrightarrow X_1 \subseteq X_2, (A_1, B_1, C_1) \supseteq (A_2, B_2, C_2)$

Combined with the above definitions, the lattice can be constructed by all approximate concepts from the {1,0,-1}-valued formal context with partial order relation is called the approximate concept lattice, denoted as $AL(K)$.

Theorem 2: A {1,0,-1}-valued formal context as a quadruple K=(U, V, {1, 0, -1}, R), $X \in 2^U$, $(A, B, C) \in 2^V \times 2^V \times 2^V$, and $(X_1, (A_1, B_1, C_1))$, $(X_2, (A_2, B_2, C_2))$ be two approximate concepts, the following property holds:

(1)$X \subseteq X^{\triangleright\triangleleft}$,$(A, B, C) \subseteq (A, B, C)^{\triangleleft\triangleright}$

(2)$X = X^{\triangleright\triangleleft\triangleright}$, $(A, B, C) = (A, B, C)^{\triangleleft\triangleright\triangleleft}$

(3)$X \subseteq (A, B, C)^{\triangleleft} \Leftrightarrow (A, B, C) \subseteq X^{\triangleright}$

(4)$X_1 \subseteq X_2 \Leftrightarrow X_1^{\triangleright} \supseteq X_2^{\triangleright}$, $(A_1, B_1, C_1) \subseteq (A_2, B_2, C_2) \Leftrightarrow (A_1, B_1, C_1)^{\triangleleft} \supseteq (A_2, B_2, C_2)^{\triangleleft}$

(5)$(X_1 \cup X_2)^{\triangleright} \Leftrightarrow X_1^{\triangleright} \cap X_2^{\triangleright}$,$((A_1, B_1, C_1) \cup (A_2, B_2, C_2))^{\triangleleft} \Leftrightarrow (A_1, B_1, C_1)^{\triangleleft} \cap (A_2, B_2, C_2)^{\triangleleft}$

Proof: An approximate concept is an extension of a classical concept, and the inclusion, intersection, concatenation, and partial order relations of its extents and connotations are logically consistent with those of the classical concepts, so the above property holds according to Theorem 1.

Theorem 3: A {1,0,-1}-valued formal context as a quadruple K=(U, V, {1, 0, -1}, R), for two approximate concepts $(X_1, (A_1, B_1, C_1))$, $(X_2, (A_2, B_2, C_2))$, the supremum and the infimum are defined as respectively:

for supremum, $(X_1, (A_1, B_1, C_1)) \vee (X_2, (A_2, B_2, C_2)) = ((X_1 \cup X_2)^{\triangleright\triangleleft}, (A_1, B_1, C_1) \cap (A_2, B_2, C_2))$

for infimum , $(X_1, (A_1, B_1, C_1)) \wedge (X_2, (A_2, B_2, C_2)) = (X_1 \cap X_2, ((A_1, B_1, C_1) \cup (A_2, B_2, C_2))^{\triangleleft\triangleright})$

Proof: We prove that the supremum and the infimum duality is verifiable. It is necessary to prove that the right equation is an approximate concept, that the right equation is the supremum of the two left concepts, and that the right equation is the smallest supremum of the two left concepts.

(i). Because $(X_1 \cup X_2)^{\triangleright\triangleleft\triangleright} = (X_1 \cup X_2)^{\triangleright} = (A_1, B_1, C_1) \cap (A_2, B_2, C_2)$, $((A_1, B_1, C_1) \cap (A_2, B_2, C_2))^{\triangleleft} = (X_1^{\triangleright} \cap X_2^{\triangleright})^{\triangleleft} = (X_1 \cup X_2)^{\triangleright\triangleleft}$, so that the right equation is an approximate concept.

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(ii). Due to $(A_1, B_1, C_1) \cap (A_2, B_2, C_2) \subset (A_1, B_1, C_1)$, $(A_1, B_1, C_1) \cap (A_2, B_2, C_2) \subset (A_2, B_2, C_2)$, so $(X_1, (A_1, B_1, C_1)) \leqslant ((X_1 \cup X_2)^{\triangleright\triangleleft}, (A_1, B_1, C_1) \cap (A_2, B_2, C_2))$, $(X_2, (A_2, B_2, C_2)) \leqslant ((X_1 \cup X_2)^{\triangleright\triangleleft}, (A_1, B_1, C_1) \cap (A_2, B_2, C_2))$, then, we proof that the right equation is the supremum of the two left concepts.

(iii). Assuming that the right equation is not a minimal supremum and there exists a minimal supremum $(X, X^{\triangleright})$, we have $(X, X^{\triangleright}) \leqslant ((X_1 \cup X_2)^{\triangleright\triangleleft}, (A_1, B_1, C_1) \cap (A_2, B_2, C_2))$, so $(A_1, B_1, C_1) \cap (A_2, B_2, C_2) \subseteq X^{\triangleright}$. Meanwhile, since $(X, X^{\triangleright})$ is the smallest supremum, $X^{\triangleright} \subset (A_1, B_1, C_1)$ and $X^{\triangleright} \subset (A_2, B_2, C_2)$, $X^{\triangleright} \subseteq (A_1, B_1, C_1) \cap (A_2, B_2, C_2)$. Combining the previous elements, we have $X^{\triangleright} = (A_1, B_1, C_1) \cap (A_2, B_2, C_2)$, so the right equation is the smallest supremum of the two concepts in the left equation.

In summary, the upper boundaries are confirmed, dually, the infimum can be proved. Therefore $AL(K)$ is a complete lattice.

Example 2: {1,0,-1}-valued formal context $K=(U, V, \{1,0,-1\}, R)$ is shown in Table 2, and approximate concept lattice $AL(K)$ is shown in Fig. 2. To simplify the explanation, we use (134, (*abde*, *c*, *ae*)) instead of ({1, 3, 4}, ({*a*, *b*, *d*, *e*}, {*c*}, {*a*, *e*})) when there is no ambiguity.

Table 2. {1,0,-1}-valued formal context *K*

	a	*b*	*c*	*d*	*e*
1	1	0	0	0	1
2	0	-1	1	1	0
3	1	1	-1	1	1
4	1	1	0	1	1

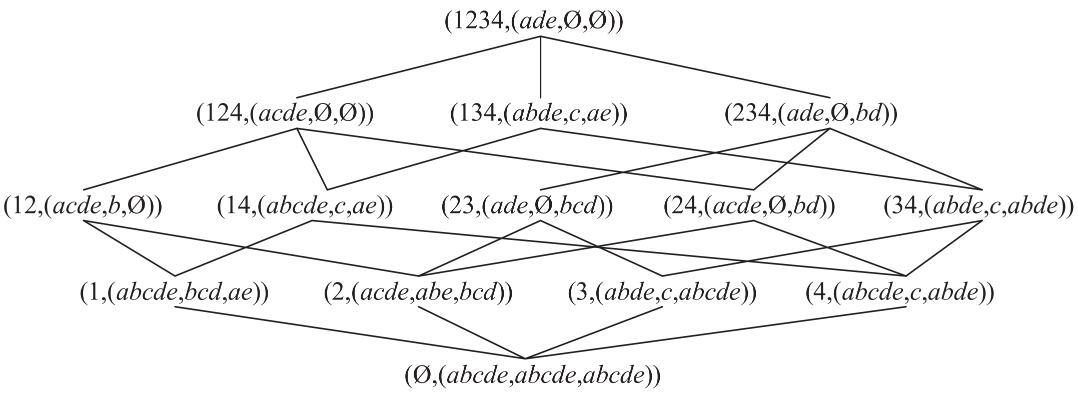

Figure 2. Approximate concept lattice $AL(K)$

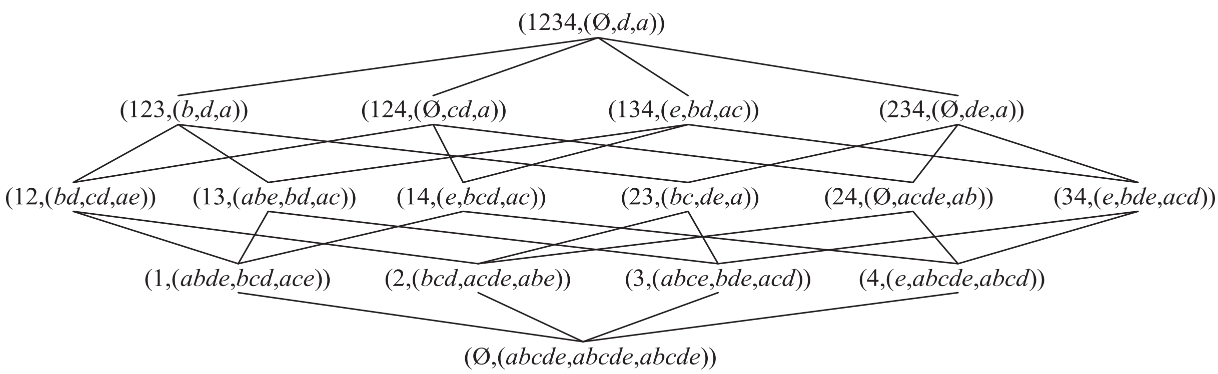

Figure 3. Approximate concept lattice $AL(K)$

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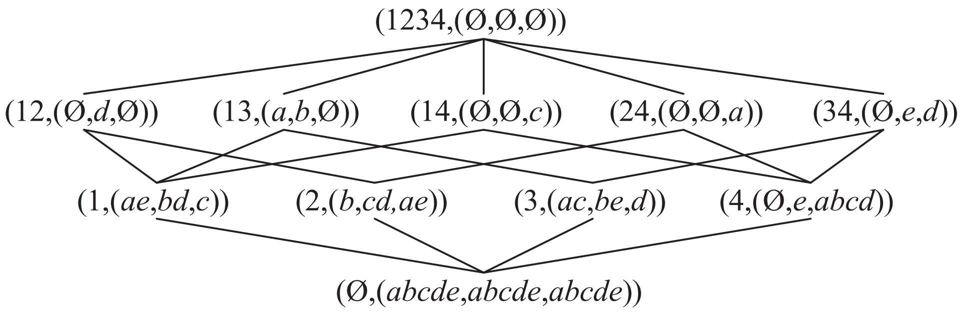

Figure 4. Three-valued concept lattice $TVL(K)$

4. COMPARISON WITH EXISTING CONCEPTS

To further demonstrate the effectiveness and distinct advantages of the proposed approximate three-way concept,this section presents a systematic comparison with three-valued concepts [17].

We examine the expressive power, flexibility in handling {1,0,-1}-valued information, and applicability to real-world data analysis tasks of each approach. Through both theoretical analysis and illustrative examples, we highlight the key distinctions between our model and existing frameworks, providing deeper insights into the enhanced knowledge representation and reasoning capabilities afforded by the approximate three-way concept.

First of all, we introduce the definition of three-valued concepts [17].

Definition 9: A {1,0,-1}-valued formal context as a quadruple $K=(U, V, \{1, 0, -1\}, R)$,for $X \in 2^U$, $X \rightarrow 2^V$, three-valued operators are defined as follow:

$X^{\#+}=\{a \in V \mid R(x, a)=1\}$

$X^{\#\sim}=\{a \in V \mid R(x, a)=0\}$

$X^{\#-}=\{a \in V \mid R(x, a)=-1\}$

The rest of the definitions of the three-value concepts can refer to the three approximate concepts given, and will not be repeated here.

the lattice can be constructed by all concepts $(X, (X^{\#+}, X^{\#\sim}, X^{\#-}))$ with partial order relation is called the three-valued concept lattice, denoted as $TVL(K)$.

Proposition 1: A {1,0,-1}-valued formal context as a quadruple $K=(U, V, \{1, 0, -1\}, R)$, for $X \subseteq U$, The following conclusions hold:

$X^{\#+}=X^{\$+} \cap X^{\$\sim}$

$X^{\#\sim}=X^{\$+} \cap X^{\$-}$

$X^{\#-}=X^{\$} \cap X^{\$\sim}$

Proof: Let $K=(U, V, \{1,0,-1\}, R)$ be an {1,0,-1}-valued formal context, $X \subseteq U$. Based on the definition of three-valued concepts and approximate three-way concepts, for a three-valued concepts $(X, (X^{\#+}, X^{\#\sim}, X^{\#-}))$ and an approximate three-way concept $(X, (X^{\$+}, X^{\$-}, X^{\$\sim}))$, we have $X^{\#+} \subseteq X^{\$+}$, $X^{\#+} \subseteq X^{\$\sim}$, $X^{\#\sim} \subseteq X^{\$+}$, $X^{\#\sim} \subseteq X^{\$-}$, $X^{\#-} \subseteq X^{\$-}$, $X^{\#-} \subseteq X^{\$\sim}$, therefore we can obtain the intent of three-valued concepts from the intent of approximate three-way concepts.

It should be pointed out that this conclusion cannot be reversed. That is, the contents of the three approximate concepts cannot be obtained through the three-value concept. For $x \in U$, a, $b \in V$, if $\{a, b\} \subseteq x^{\$+}$, Then, the values of $R(x, a)$ and $R(x, b)$ have the following three cases: $R(x, a)=R(x, b)=1$, $R(x, a)=R(x, b)=0$, or $R(x, a)=1$ and $R(x, b)=0$, (or $R(x, a)=0$ and $R(x, b)=1$), in three-valued concepts, $X^{\#+}$ and $X^{\#\sim}$ can represent case 1 and case 2, but can not represent case 3, therefore we can't obtain $X^{\$+}$ from $X^{\#+}$ and $X^{\#\sim}$.

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In summary, the approximate three-way concept contains the information described by the three-valued concept. Since Proposition 1 cannot be reversed, it is impossible to obtain additional information about the approximate three-way concept through the three-valued concept.

Example 3: {1,0,-1}-valued formal context K=(U, V, {1,0,-1}, R) is shown in Table 3, approximate lattice $AL(K)$ and three-valued concept lattice $TVL(K)$ is shown in Fig. 3 and Fig. 4, respectively.

Table 3. {1,0,-1}-valued formal context K

	a	*b*	*c*	*d*	*e*
1	1	0	-1	0	1
2	-1	1	0	0	-1
3	1	0	1	-1	0
4	-1	-1	-1	-1	0

Similar to Example 2, to simplify the explanation, we use (2, (b, cd, ae))instead of ({2}, ({b},{c, d}, {a, e})) when there is no ambiguity.

Comparing the two concept lattices, we can see that $\{1, 3\}^{\#+}=\{1, 3\}^{\$+}\cap\{1, 3\}^{\$\sim}=\{a, b, e\}\cap\{a, c\}=\{a\}$, but $\{1, 3\}^{\$+}=\{a, b, e\} \neq \{1, 3\}^{\#+}\cup\{1, 3\}^{\#\sim}=\{a, b\}$, that is because $R(1, e)$=1 and $R(3, e)$=0, three-valued concepts can't express this situation.

5. CONCLUSION

In many real-world applications, information is often characterized by uncertainty and {1,0,-1}-valuedness, making it challenging for traditional formal concept analysis to effectively capture such complexity. To address this limitation, this paper incorporates the notion of {1,0,-1}-valuedness into the formal context framework and explores an extended concept model tailored for this setting. The rationality and effectiveness of the proposed concept extension have been validated through its application to tasks such as information fusion and conflict analysis.

By accommodating partial and uncertain information, {1,0,-1}-valued formal contexts allow for a richer and more nuanced representation of the relationships between objects and attributes. This enhancement broadens the scope of potential applications, especially in areas like knowledge representation and data mining, where flexibility and expressiveness are critical. Furthermore, the advancement of concept lattice theory need not remain confined to traditional frameworks. Its integration with related theories—such as fuzzy sets, rough sets, and granular computing—opens new avenues for tackling challenges in domains like machine learning and artificial intelligence, offering promising tools and methodologies for complex problem-solving.

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